

# The Effect of a Tangential Frictional Force on Rotating Disks: An Experimental Approach

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This paper describes an experiment with two touching rotating disks, whose movement is followed by video analysis. Within the disks' movements, there are intervals with sliding and intervals without sliding, that is, intervals with frictional forces between the touching surfaces and intervals without it. This system configuration allows for measurement of the changeable magnitudes and directions of frictional forces (much more difficult to set up with translational motion). This activity may be used to combat students' misconceptions of "frictional force always opposes the motion" and "between the same two bodies, one gets the same frictional force," commonly found in the classroom,<sup>1-3</sup> or just to reinforce the rotational dynamics relationships.

## Theoretical background

The following forces act upon the system (depicted in Fig. 1): gravitational forces, vertical, pointing to the center of Earth; action of disks on each other with a tangential component (the tangential friction forces, in the horizontal plane, tangent to the disks, opposing the relative rotational movement, equal in magnitude) and a normal component in the  $y$ -direction; reaction forces with a vertical component (canceling the gravitational force) and a horizontal component, exerted by the axles on the disks, canceling the horizontal forces.

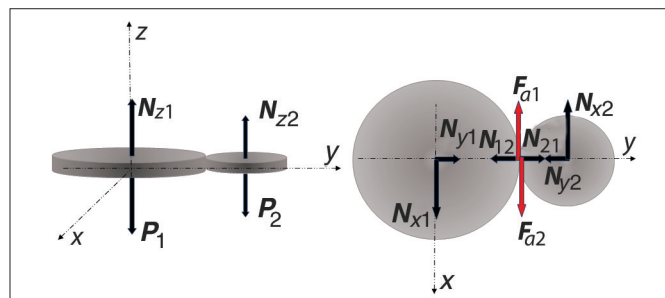


Fig. 1. Forces acting on disk 1 and disk 2.  $P_1$  is the gravitational force acting on disk 1, and  $F_{a1}$  and  $N_{12}$  are the tangential and normal forces exerted by disk 2 on disk 1.  $N_{x1}$ ,  $N_{y1}$ , and  $N_{z1}$  are the components of the normal force exerted by the axle on disk 1. In a similar way for disk 2.

If disk 1 (the big disk with radius  $R$  and moment of inertia  $I_1$ ) while rotating around the  $z$ -axis is slightly pushed against disk 2 (the small disk with radius  $r$  and moment of inertia  $I_2$ ), initially at rest, frictional forces will appear opposing the relative rotational motion. They will have different effects on the disks: the big disk will be decelerated, and the small disk will be accelerated until they rotate with equal tangent speeds. At this instant there is no relative motion between the surfaces, the tangential velocities are equal, and friction disappears. If the velocity of one of the disks suddenly drops, (this can be

achieved by using one's fist to give a quick punch on top of the disk, see Fig. 2, right), the frictional force reappears until a synchronous movement is attained again. This system is therefore an easy showcase to confront the students' misconceptions: the frictional force does not always act to *oppose the motion*; indeed, it is the frictional force that causes the small disk to rotate. And within the same situation, two disks rotating in contact, the force can appear, disappear, or even invert its direction.

Starting from the rotational dynamics equations, and noting that all forces (except friction) acting on disk 1 have zero torque with respect to the rotation axis (the same for disk 2), one can write the net torque

$$R \times F_{a1} = I_1 \cdot \alpha_1 \quad (1)$$

$$r \times F_{a2} = I_2 \cdot \alpha_2 \quad (2)$$

and relate the angular acceleration to the magnitude of frictional force.

## Experimental setup

Several components from the rotational system ME-8950A (PASCO, Roseville, CA) were used in the experimental setup. A 22.8-cm diameter disk ( $I = 0.0091 \text{ kg}\cdot\text{m}^2$ ) and a 7-cm diameter small black disk (intended to be a 900-g counterweight,

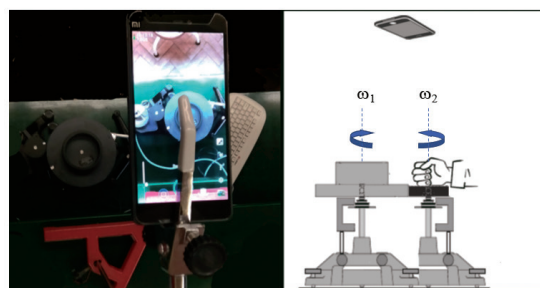


Fig. 2. Photo of the experimental setup (seen from the top) and scheme (seen from the side). The fist shows how the punch is applied to the smaller disk.

$I = 0.00063 \text{ kg}\cdot\text{m}^2$ , see Gomes et al.<sup>4</sup>) were mounted on two cast iron bases using the rotating axles included in the kit. A ring (12.7-cm outer diameter, 1420 g,  $I = 0.0048 \text{ kg}\cdot\text{m}^2$ ) was placed on top of the big disk to increase the moment of inertia. Small pieces of white tape were stuck on the periphery of the disks so as to enable rotation tracking through video analysis. Two small photogate heads were placed in such a way that their infrared beams can be interrupted by the 10-spoke pulleys on the vertical axles as they turn. A smartphone (Xiaomi Mi 4c, 30 fps) was placed half a meter above the disks.

The large disk was spun at a certain angular speed (by hand spinning of the axle below the disk) and then the small disk—

**Table I. Magnitudes of velocities, accelerations, and frictional forces (experimental uncertainties in round brackets).**

magnitude of	Slipping disks after touching	synchro-nous	First punch	synchro-nous	Second punch	synchro-nous	Third punch	synchro-nous	Fourth punch	synchronous
Angular acceleration ( $\text{rad/s}^2$ )	$\alpha_{\text{small}} = 30(1)$ $\alpha_{\text{big}} = 4.4(1)$		$\alpha_{\text{small}} = 19(1)$ $\alpha_{\text{big}} = 2.6(1)$		$\alpha_{\text{small}} = 18(1)$ $\alpha_{\text{big}} = 2.2(1)$		$\alpha_{\text{small}} = 20(1)$ $\alpha_{\text{big}} = 2.7(1)$		$\alpha_{\text{small}} = 18(1)$ $\alpha_{\text{big}} = 2.6(1)$	
Frictional Force (N)	$F_{\text{small}} = 0.55(2)$ $F_{\text{big}} = 0.54(1)$		$F_{\text{small}} = 0.35(2)$ $F_{\text{big}} = 0.32(1)$		$F_{\text{small}} = 0.33(2)$ $F_{\text{big}} = 0.27(1)$		$F_{\text{small}} = 0.35(2)$ $F_{\text{big}} = 0.33(1)$		$F_{\text{small}} = 0.33(2)$ $F_{\text{big}} = 0.31(1)$	
Peripheric velocity (m/s)		$v_{\text{small}} = 2.7(2)$ $v_{\text{big}} = 2.7(1)$		$v_{\text{small}} = 1.7(2)$ $v_{\text{big}} = 1.8(1)$		$v_{\text{small}} = 1.02(5)$ $v_{\text{big}} = 1.03(4)$		$v_{\text{small}} = 0.52(3)$ $v_{\text{big}} = 0.52(3)$		$v_{\text{small}} = 0.22(2)$ $v_{\text{big}} = 0.21(2)$

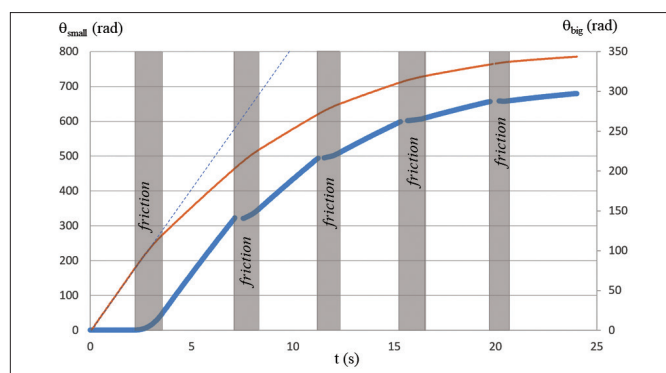
initially at rest—was approached till they touched. At intervals, a punch was given on the small disk, to force an abrupt decrease of its speed (Fig. 2).

The video file was then handled using Tracker—a free open source video analysis and modeling tool—the positions of the white tapes were retrieved, and the data were exported to a CSV file, for further data processing and plotting (in an MS Excel spreadsheet).

## Experimental results

The values obtained from preliminary runs from the video analysis and from photogate and PASCO software matched perfectly, so that in subsequent runs only video analysis was used.<sup>5</sup>

Figure 3 shows the variation with time of the angular position of both disks. Both lines exhibit linear sections that correspond to constant tangential speed without slippage



**Fig. 3.** Plot of  $\theta$  as a function of  $t$  for the two disks. The angular position of the big disk, disk 1, is plotted with thin orange markers/lines. The angular position of the small disk, disk 2, is plotted with thick blue markers/lines. The intervals in which there is slippage between the disks are highlighted in grey. A dashed line shows the projected angular position for the big disk if there were no frictional forces. Some points are missing due to the obstruction of the video camera by the punching fist.

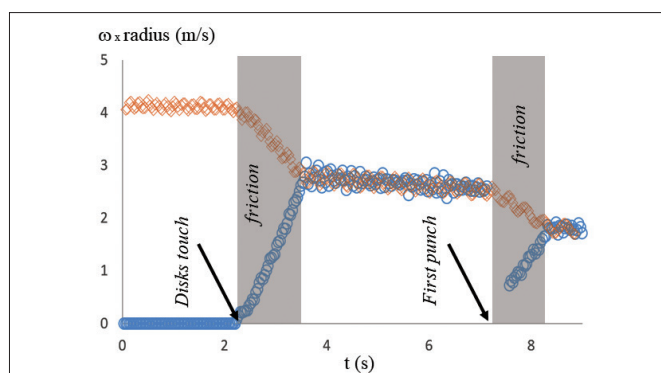
and parabolic sections where there is a tangential acceleration (acceleration in opposite directions as evidenced by the opposite nature of the curvature of the two lines in Fig. 3). By fitting the curve with a second-order polynomial function in the slippage stage, one can extract the value of the angular acceleration, and—from that—the magnitude of the frictional force. By fitting of the curves in the synchronous, non-slippage stages, with a linear fit, one retrieves the values of the

angular velocities. Table I summarizes the results obtained for the several stages, using Eqs. (1) and (2) and, as expected, the modulus of the pair of frictional forces is identical during slippage (as required by Newton's third law) and the tangential speed of both disks is equal and constant during the synchronous non-slippage stages.

The data in Table I can be visualized in Fig. 4, where the variation of the tangential velocity of both the disks is plotted. Once the disks touched, there was an acceleration of the small disk and a deceleration of the big disk until the contact points moved with the same velocity.

At that instant, the friction force disappeared, and the two disks stayed with constant velocities. When the first punch disturbed the system, the frictional force reappeared causing new opposing accelerations.

In conclusion, this activity, as described above, allows the teacher to set an example that challenges common student



**Fig. 4.** Plot of angular speed  $\times$  radius as a function of  $t$  for the two disks, in the 0–9-s interval (disk 1- thin orange markers, disk 2- thick blue markers). The numerical values were obtained by smoothing the numerical derivative of  $\theta(t)$ . The intervals in which there was slippage between the disks are highlighted in grey.

misconceptions about friction. The use of the video analysis allows the user to eschew sensors, interfaces, and dedicated software, and proves again to be a reliable tool in mechanics physics courses.<sup>6-13</sup>

## Acknowledgments

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## And the Survey Says ...

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### What are typical starting salaries for new physics PhDs?

We have seen that physics PhD recipients follow a variety of paths upon completing their degree. Those who accept potentially permanent positions in the private sector typically report higher starting salaries. Among postdocs, those working at government labs typically report higher starting salaries than those working at universities and UARI (University Affiliated Research Institutes). It is possible that some of the difference there owes to 9/10-month contracts vs. 11/12-month contracts.

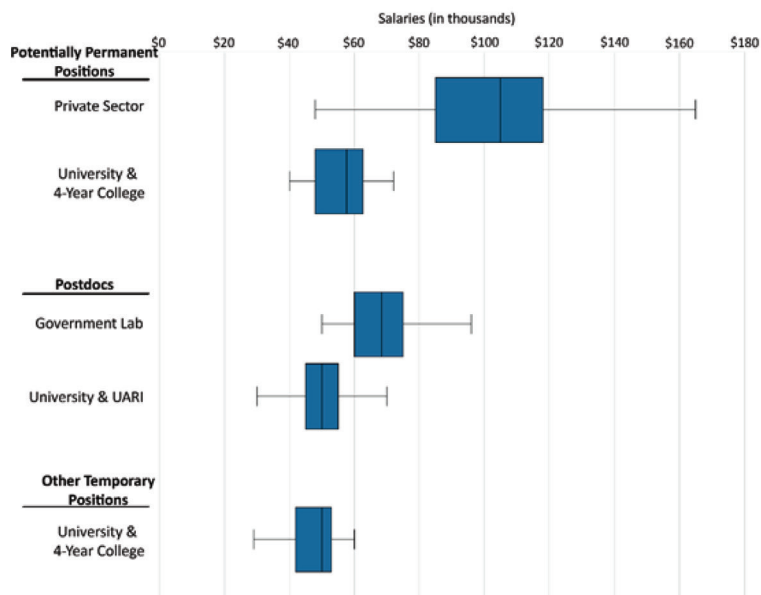
In May, we will look at where these physics PhD recipients hope to be working in 10 years.

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**Starting Salaries for New Physics PhDs, Classes of 2015 & 2016 Combined**



Data represents only U.S.-educated PhDs who remained in the U.S. after earning their degrees. The full starting salary range is represented by the lines extending to each side of the box. The box represents the middle 50% (25th to 75th percentile) of the salaries. The vertical line within the box represents the median starting salary for the sector. Government Lab included federally funded research and development centers; e.g., Los Alamos National Laboratory. UARI is a university affiliated research institute. The data for PhDs holding potentially permanent positions in academia include salaries based on 9-10 and 11-12 month commitments and have not been adjusted. Data are based on respondents holding potentially permanent positions in the private sector (214) and in universities and 4-year colleges (30), postdocs in government labs (78) and universities and UARIs (257), and "other temporary positions" in universities and 4-year colleges (24).

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